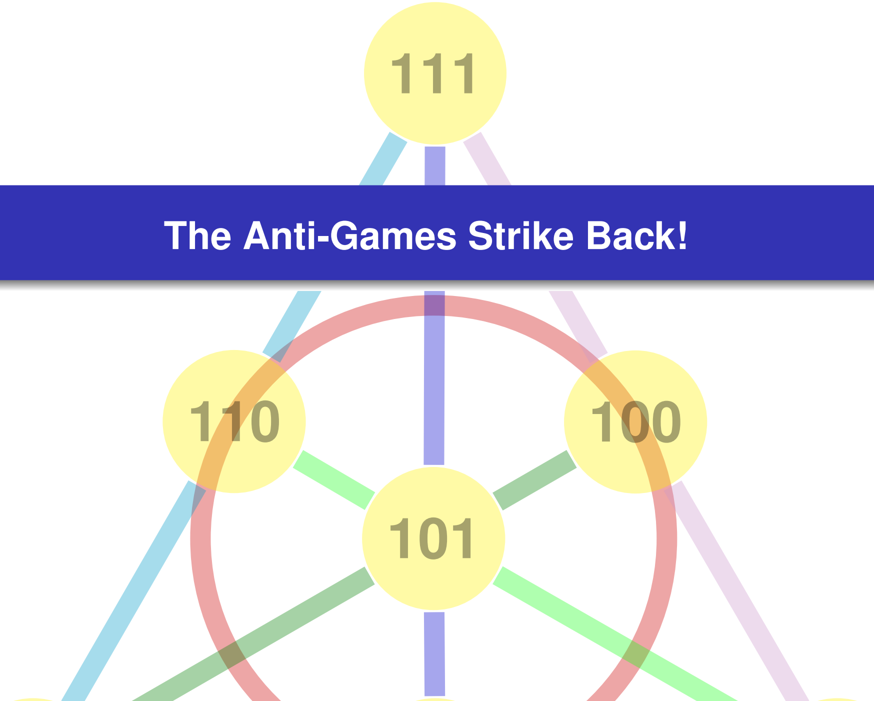
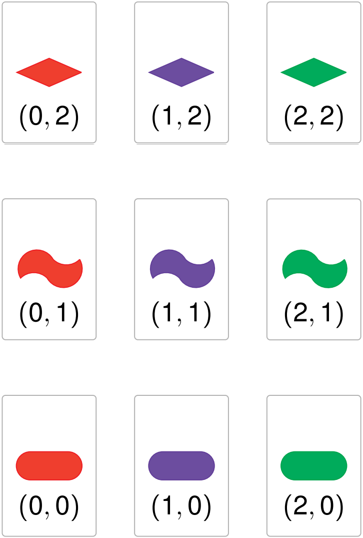
The Anti-Games Strike Back!



Let's play Anti-SET! ($AG(2)$)

First to get a set loses.



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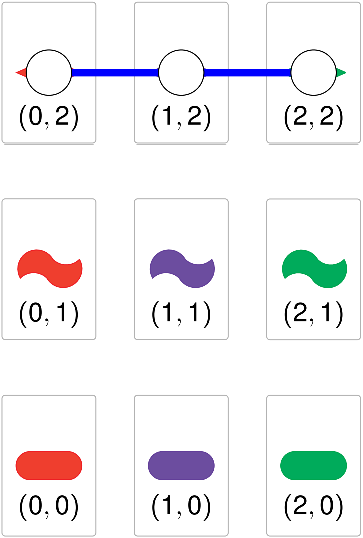

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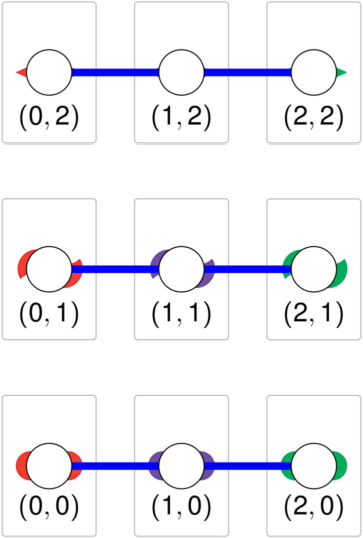

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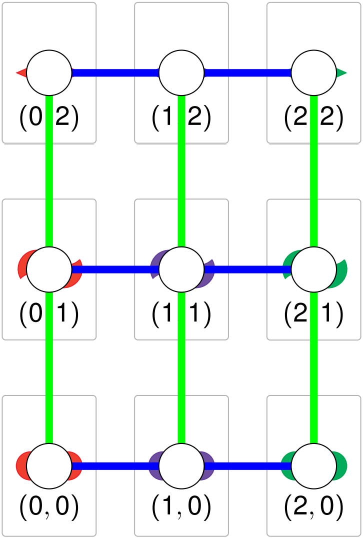

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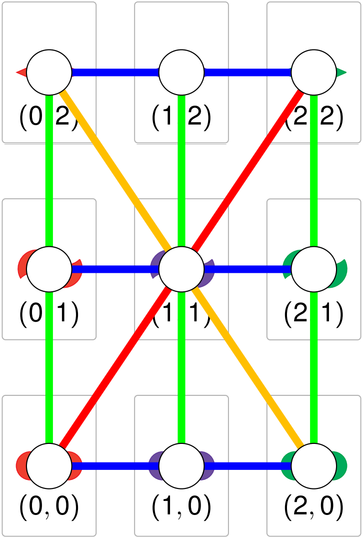

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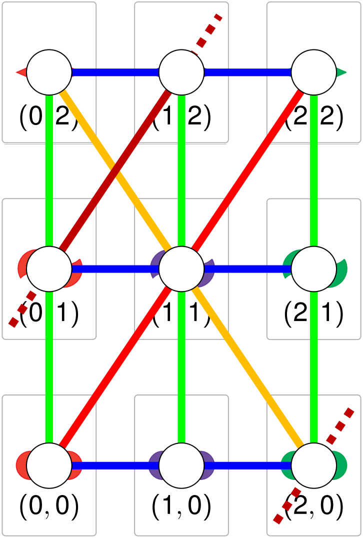

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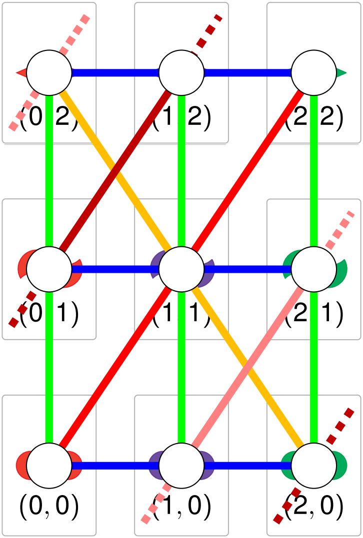

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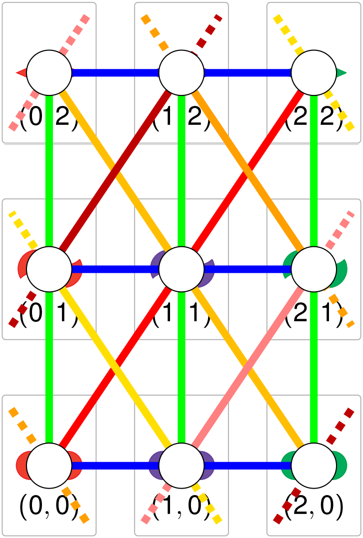
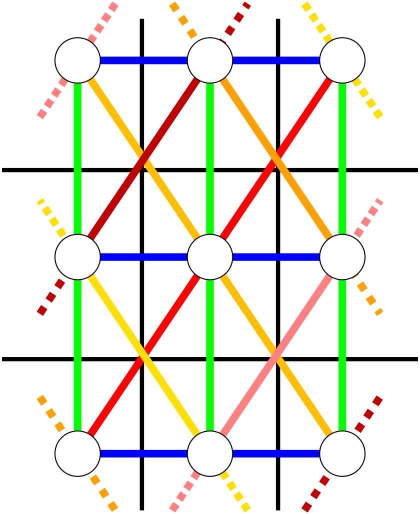


Let's play Anti-Tic-Tac-Toe!

First to get 3 in a row loses.



Anti-Tic-Tac-Toe

Anti-Tic-Tac-Toe is a game with two players:

Xavier (1st)   and   Olivia (2nd).

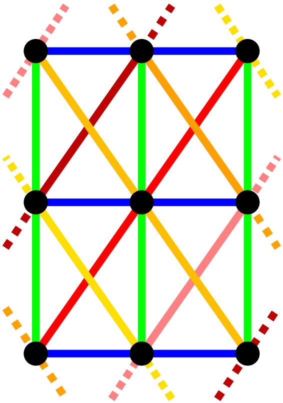
- The game is played on a “board” of points and lines.
- $\mathcal{X}$ and $\mathcal{O}$ alternate marking any unmarked point.
- A player loses immediately if they own a line.

The fun depends on the “board”

Jacob Van Hook & Sophia Mancini
Mathfest 2017

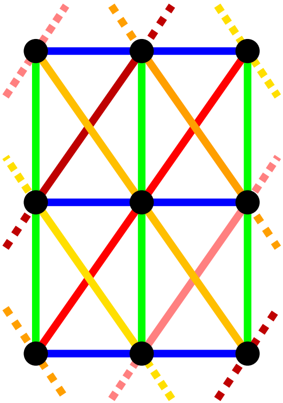
What makes a good anti-tic-tac-toe board?

- 3 points per line.
- For every pair of points **there is** a **unique** line containing them.
- “Enough” points to avoid ties (> 3 points)



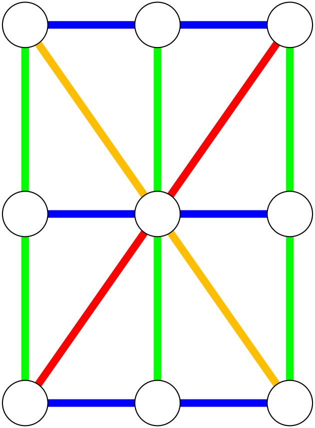
What makes a good anti-tic-tac-toe board? Steiner* Triple Systems!

- 3 points per line.
- For every pair of points **there is** a **unique** line containing them.
- "Enough" points to avoid ties ($>$ 3 points)



* Stigler's law

Steiner triple system or not?



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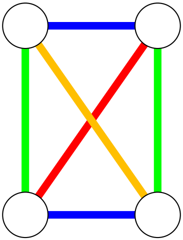

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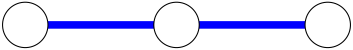
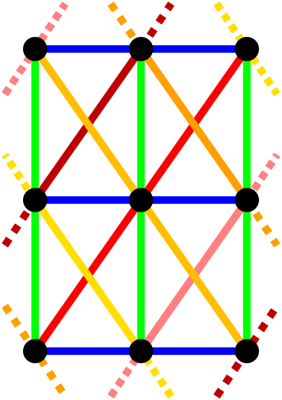


Last time . . .



We showed that **Xavier wins** anti-tic-tac-toe on **affine Steiner Triple Systems (AGs)**

What other STSs are there?

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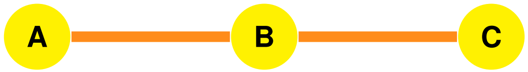
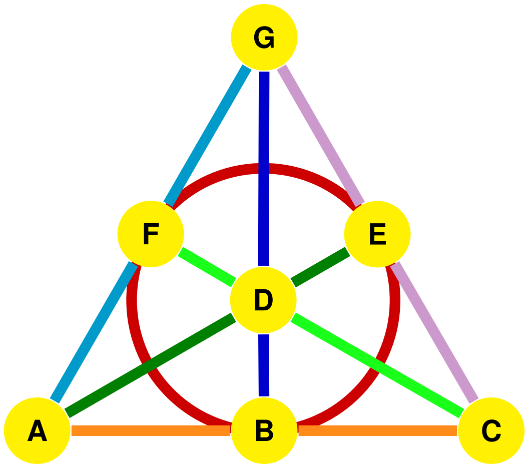


Projective Geometry

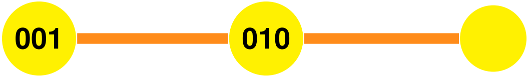
Another kind of Steiner Triple System



Projective Geometry $PG(3)$

Points: Nonzero binary points (p_1, p_2, p_3)

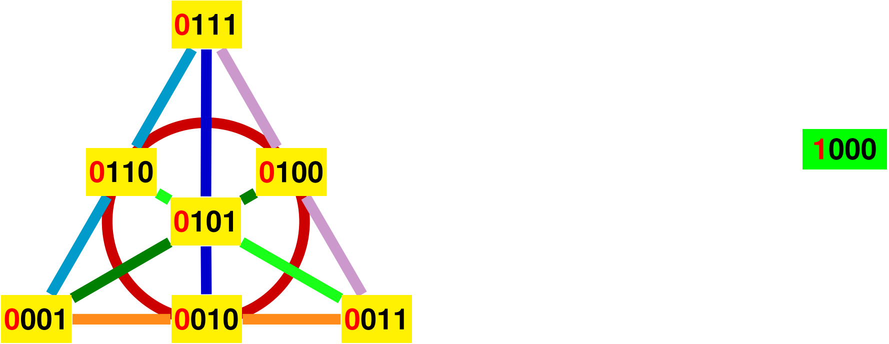
Lines: $\ell + m \equiv n \pmod 2$



Projective Geometry $PG(4)$

Points: Nonzero binary points (p_1, p_2, p_3, p_4)

Lines: $\ell + m \equiv n \pmod 2$



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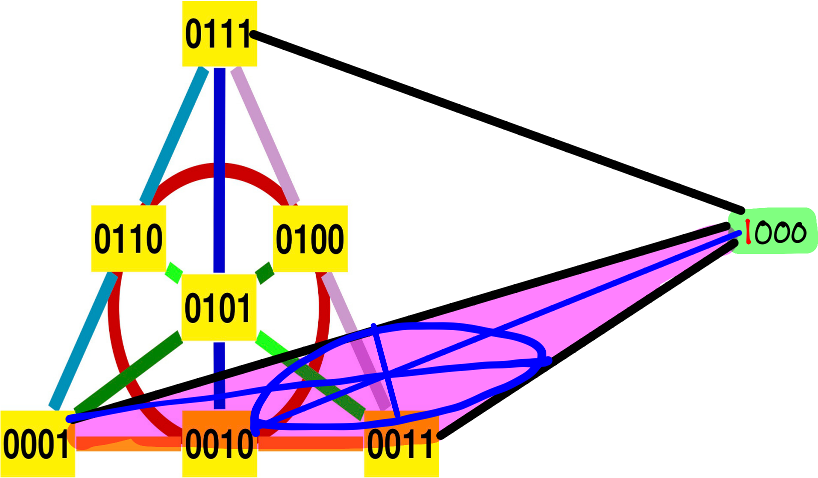

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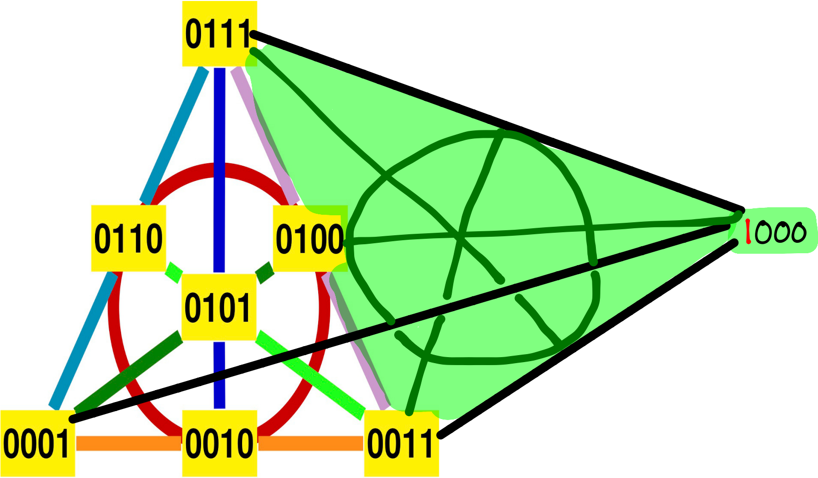

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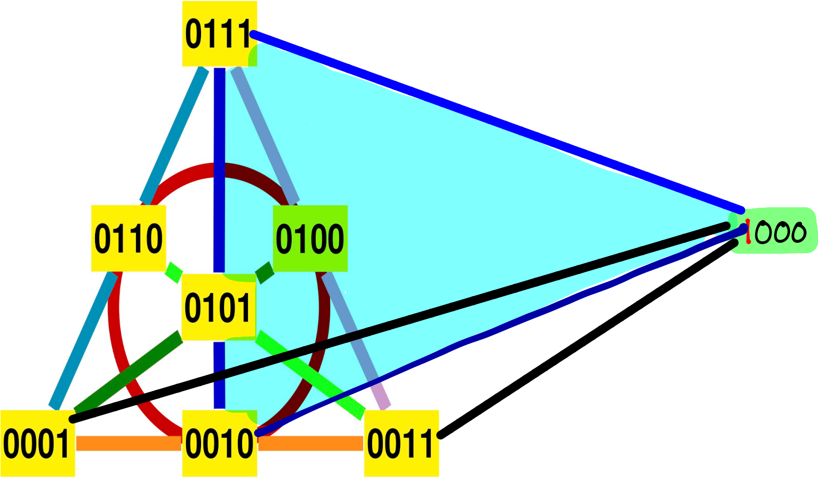

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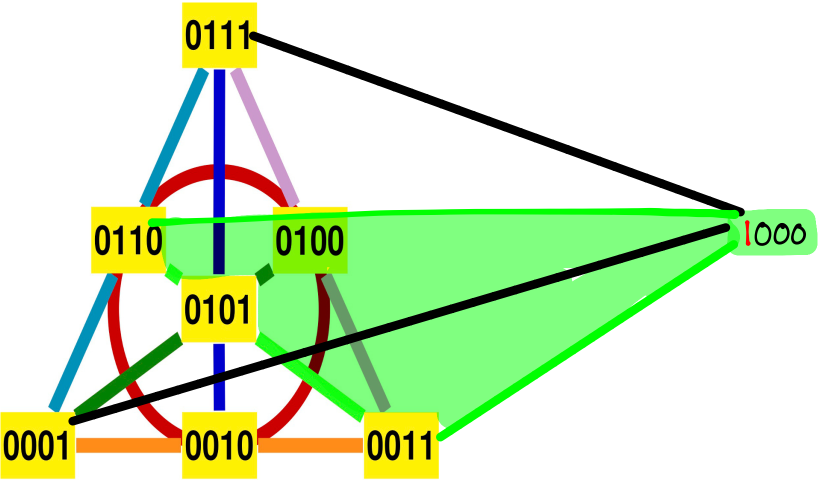

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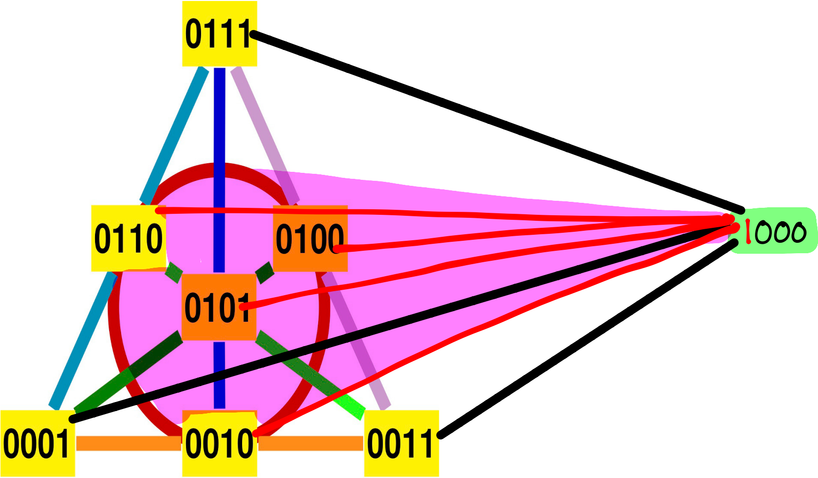
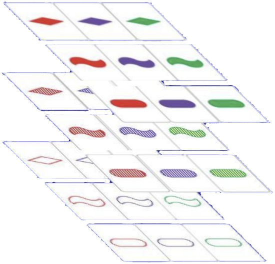
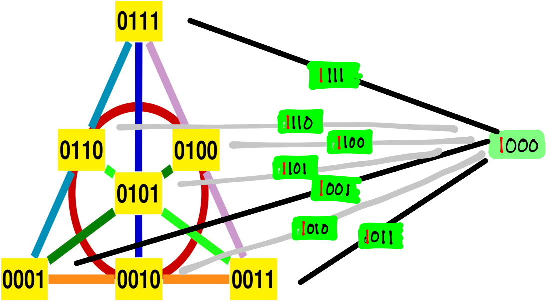


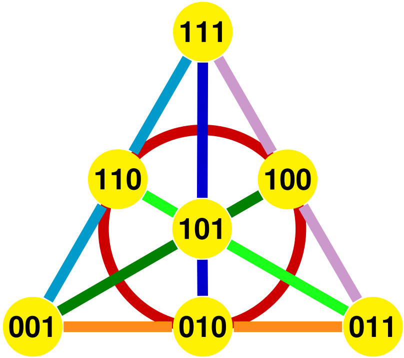
Projective Geometries "grow" much like Affine Geometries





Caps

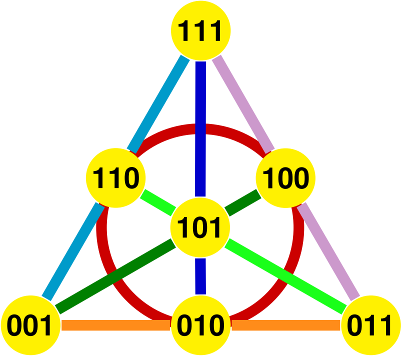
What's the *one* special feature of $\mathcal{X}$'s and $\mathcal{O}$'s points?



Caps

What's the *one* special feature of $\mathcal{X}$'s and $\mathcal{O}$'s points?

Cap: A set of points that contains no line.



PG(3) Max cap:

Rest is

Caps

What's the *one* special feature of $\mathcal{X}$'s and $\mathcal{O}$'s points?

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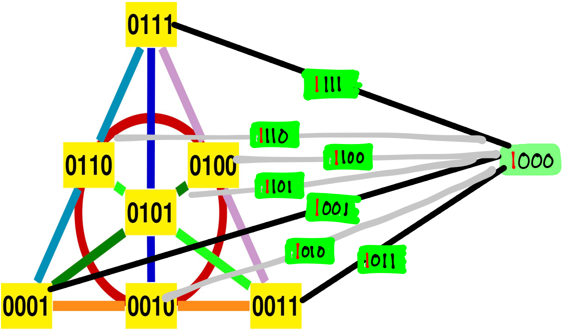

PG(3) Max cap: 2^2
Rest is *PG*(2)

PG(4) Max cap:
Rest is

Caps

What's the *one* special feature of $\mathcal{X}$'s and $\mathcal{O}$'s points?

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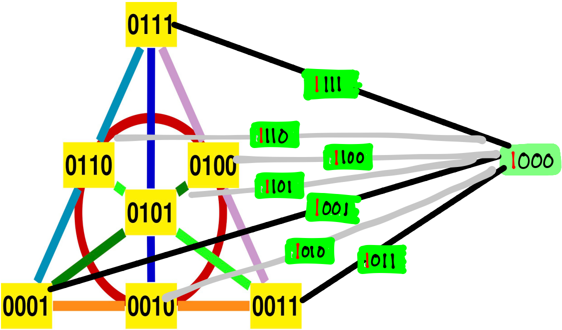
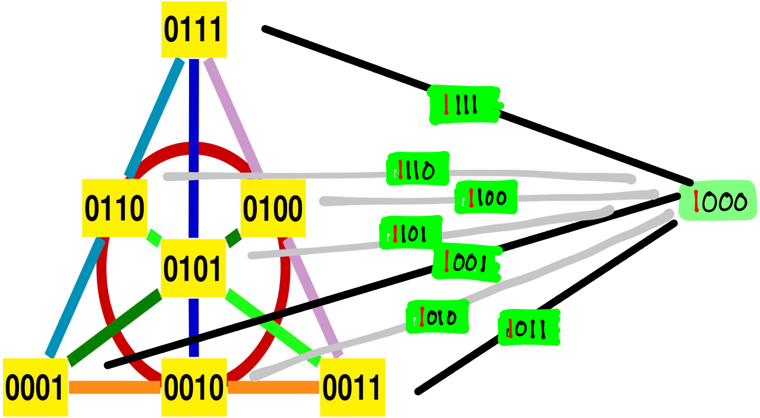


PG(3) Max cap: 2^2
Rest is *PG*(2)

PG(4) Max cap: 2^3
Rest is *PG*(3)

PG(*n*) Max cap:
Rest is

How could we win Anti-Tic-Tac-Toe on $PG(n)$?



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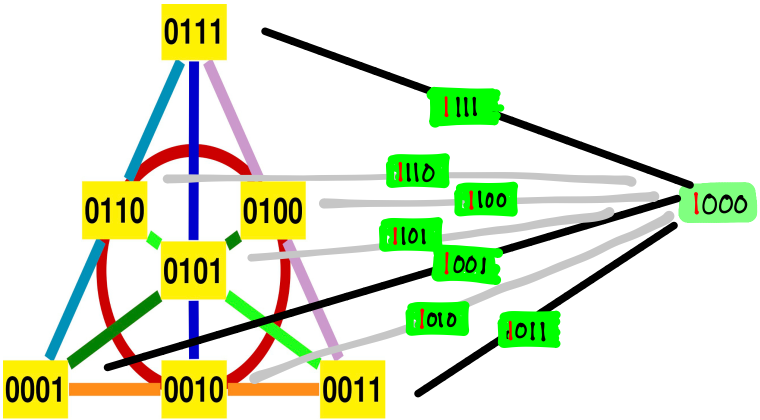
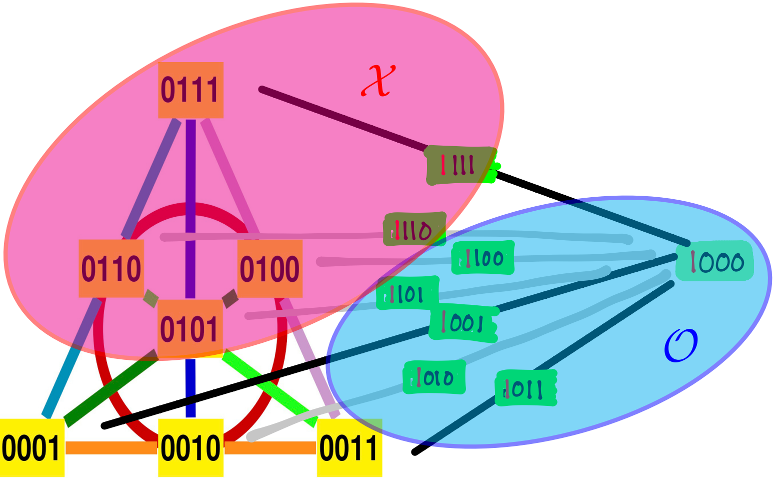


Each player could (possibly) take: $\frac{3}{4}2^{n-1}$ ($\frac{3}{4}$ of a maximum cap!)

How could we win Anti-Tic-Tac-Toe on $PG(n)$?



Who wins?

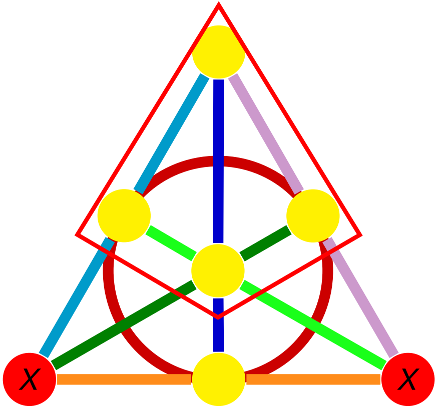
How to win Anti-Tic-Tac-Toe on $PG(n)$

Winning condition for $\mathcal{O}$

If $\mathcal{O}$ can take $\frac{3}{4}$ of the points in a max cap,
then $\mathcal{X}$ must lose the game on their next move.

Pasch tiles

A maximum cap can be *tiled*:



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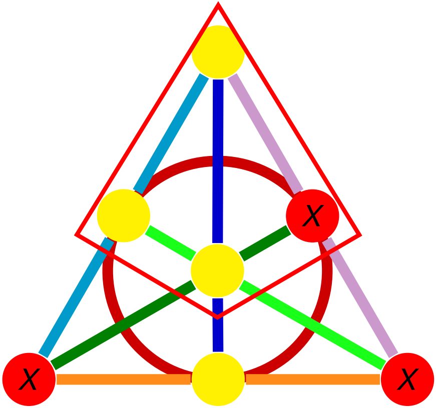

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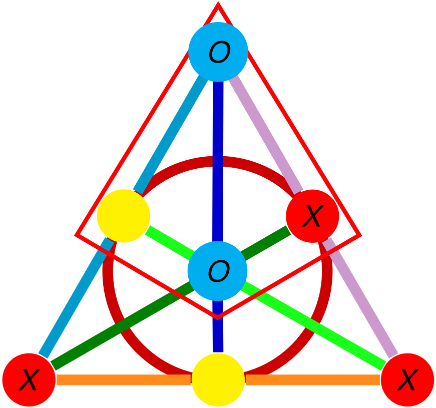

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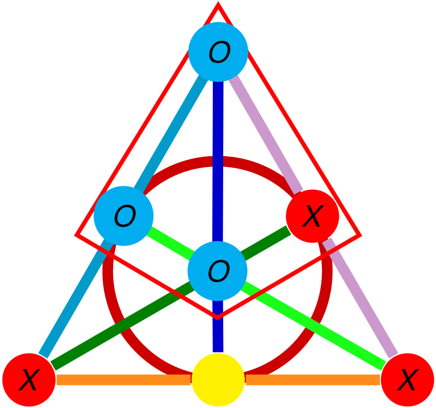

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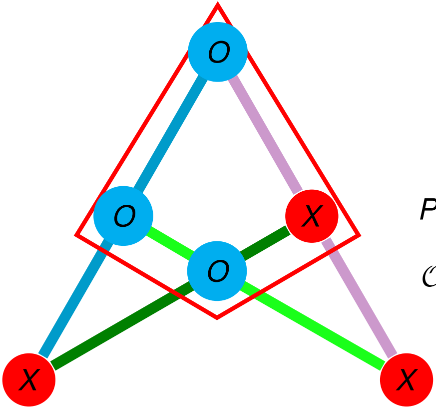
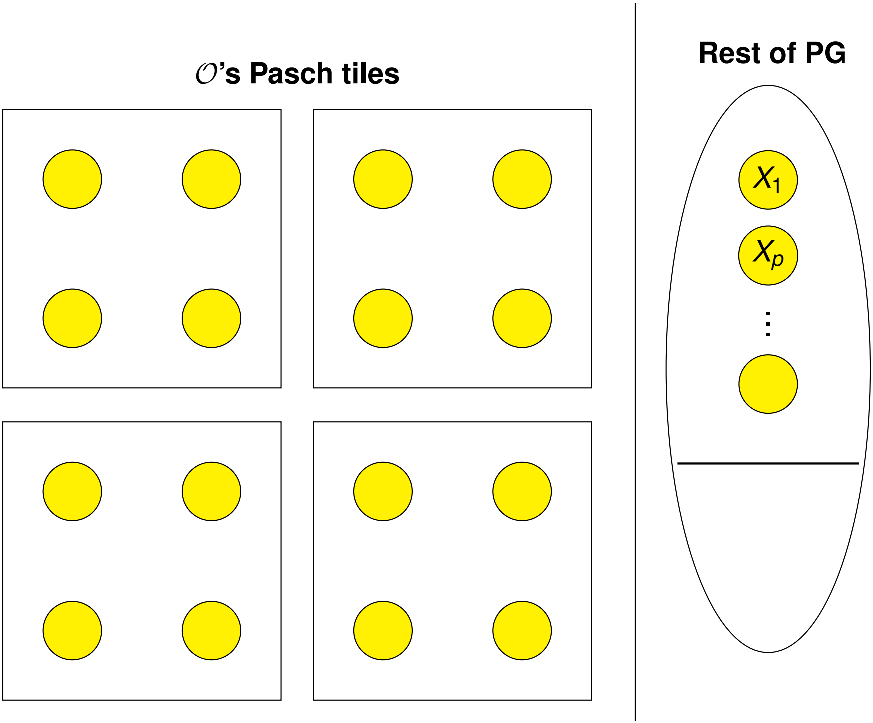
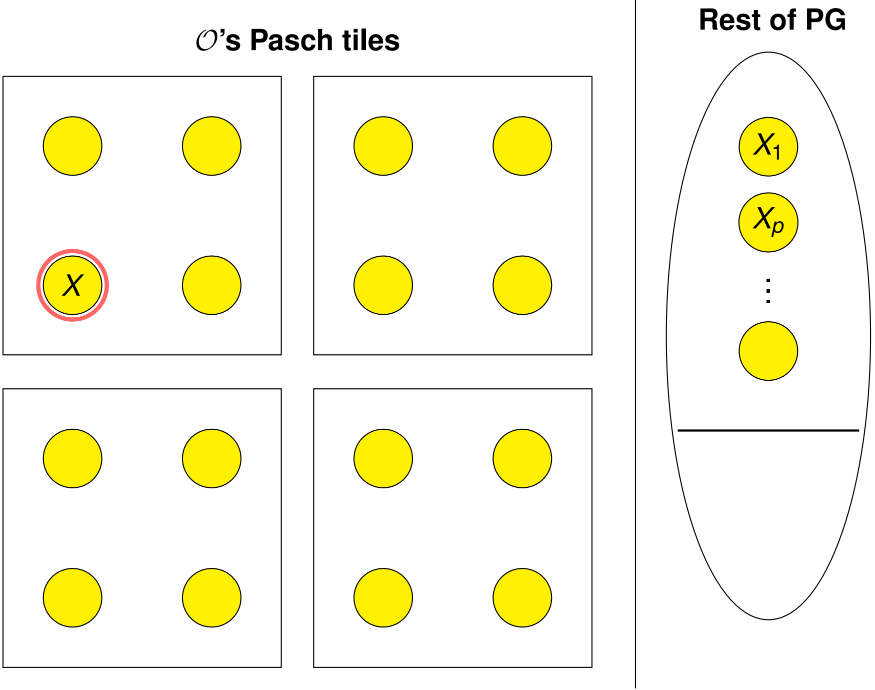


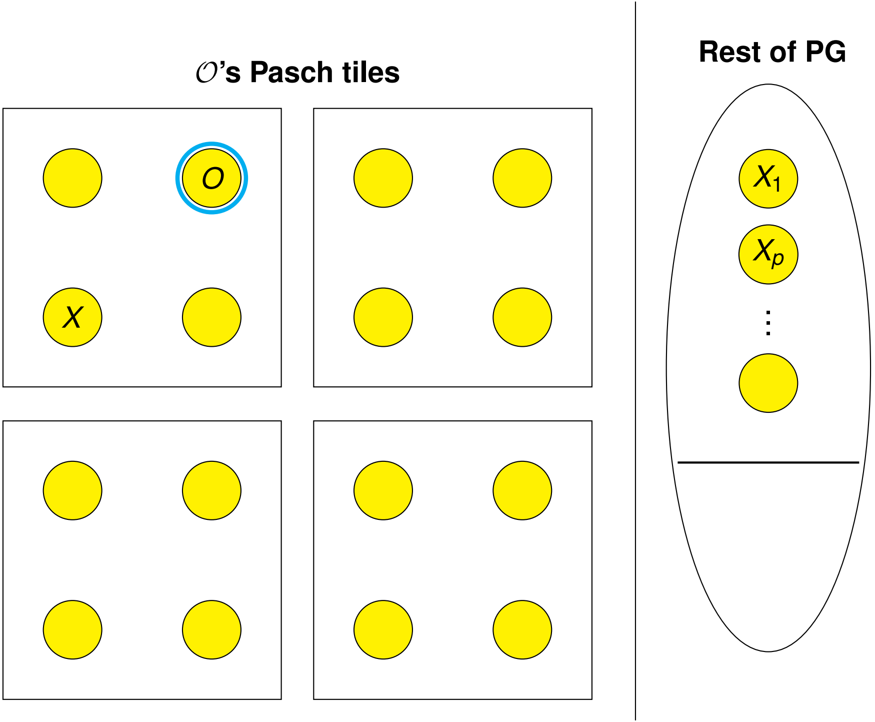
The
Pasch Configuration

$\mathcal{O}$ can take $\frac{3}{4}$ points
in each tile

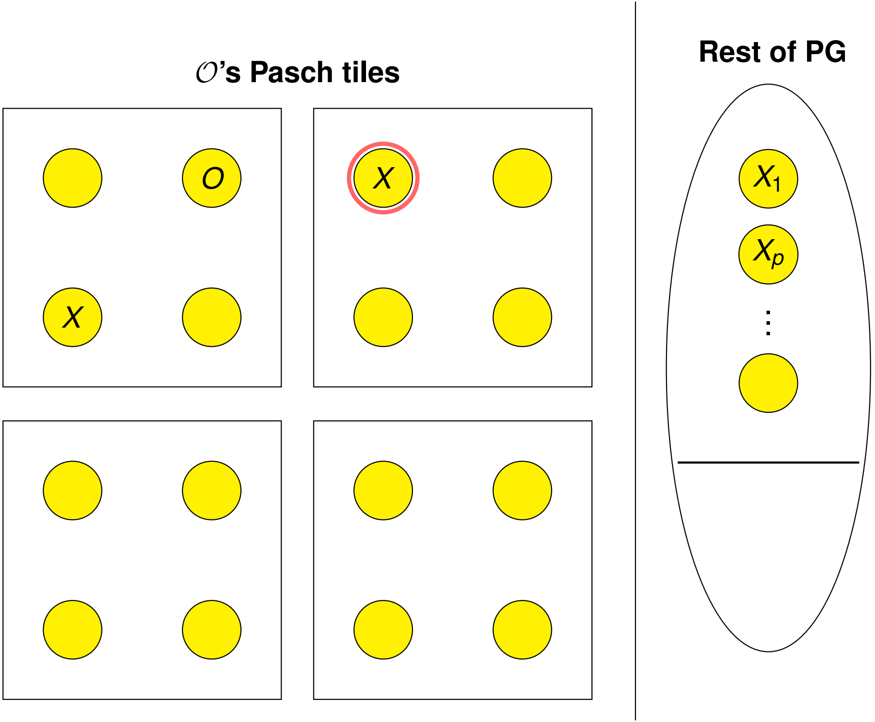

𝒪's Pasch tiles
Rest of PG
X_1
X_p
⋮

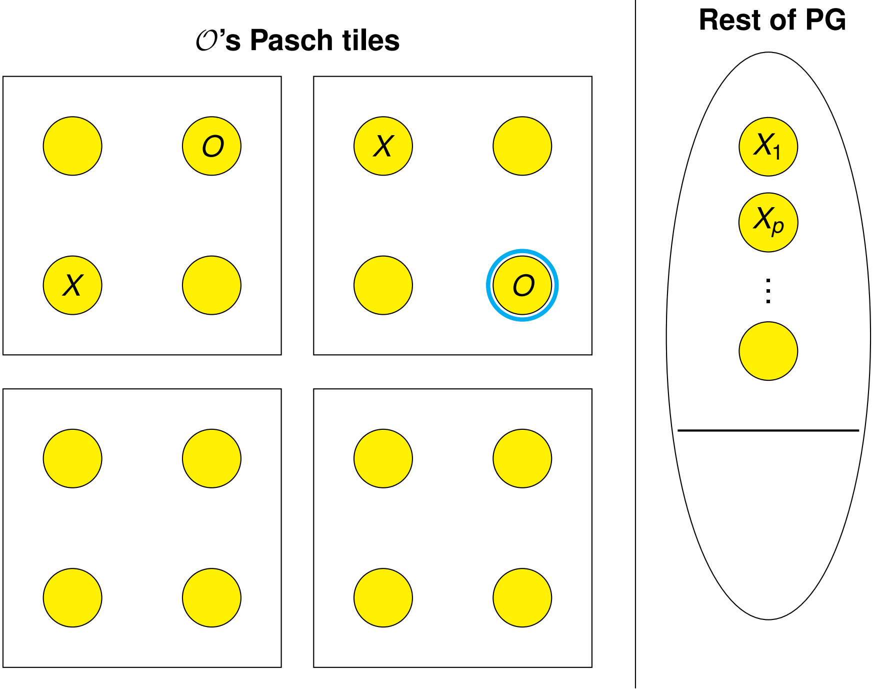


$\mathcal{O}$'s Pasch tiles
Rest of PG
X
X_1
X_p

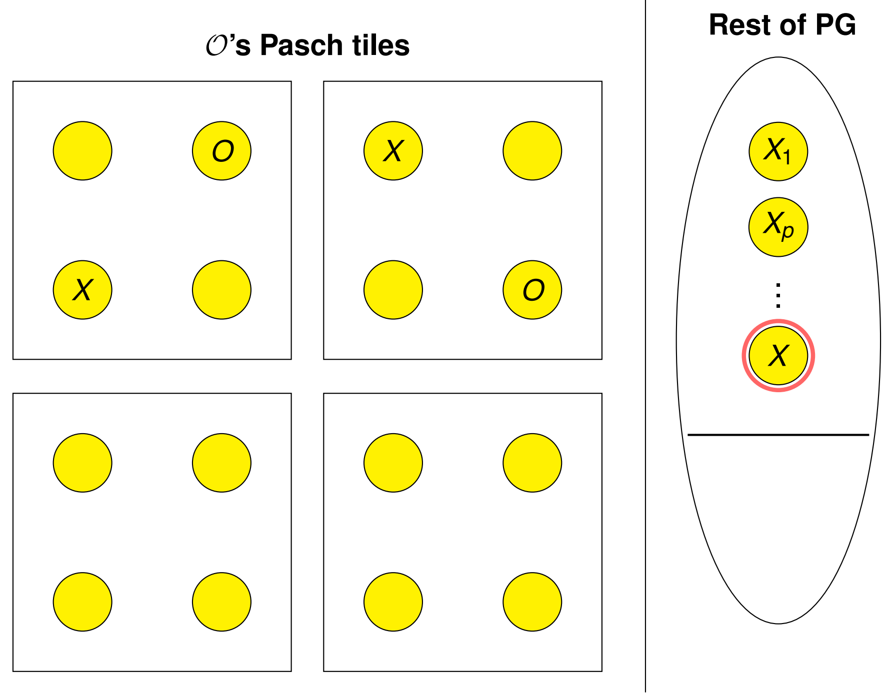


$\mathcal{O}$'s Pasch tiles
O
X
Rest of PG
X_1
X_p

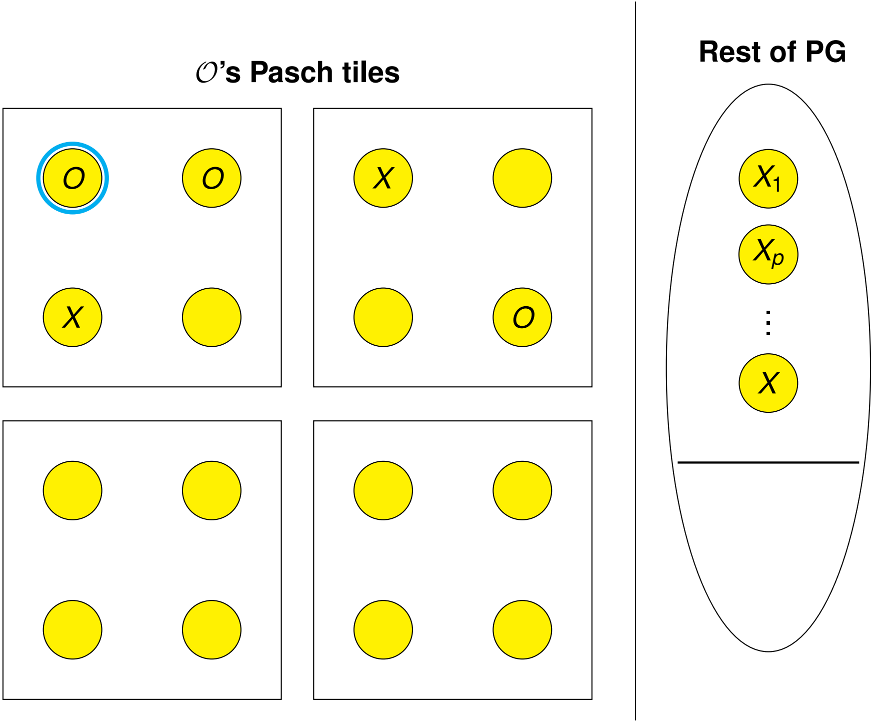

$\mathcal{O}$'s Pasch tiles
O
X
X
Rest of PG
X_1
X_p
⋮



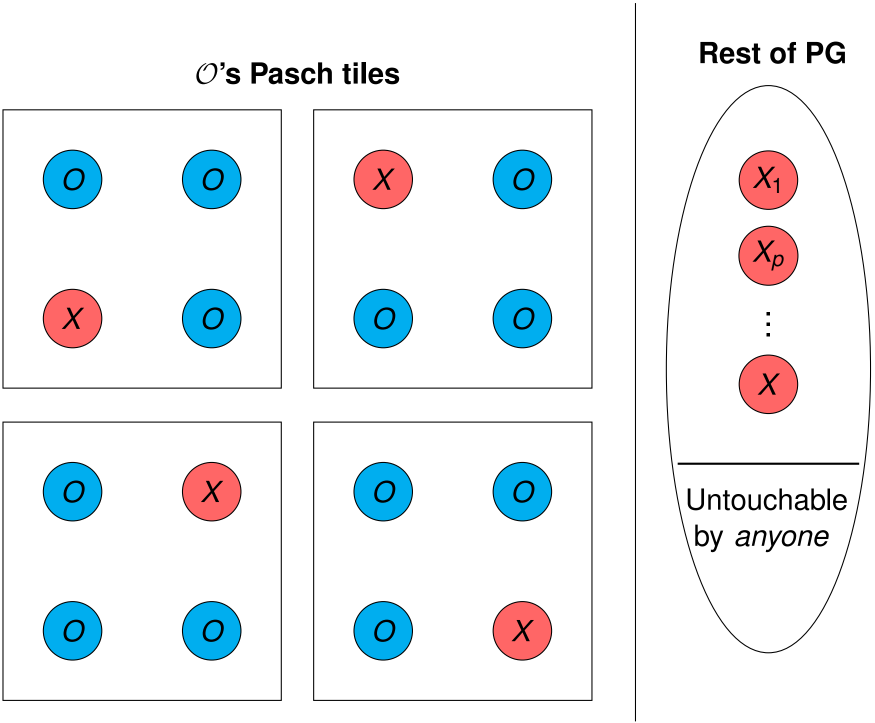
𝒪's Pasch tiles
O
X
X
O
Rest of PG
X_1
X_p



$\mathcal{O}$'s Pasch tiles
O
X
X
O
Rest of PG
X_1
X_p
⋮
X



𝒪's Pasch tiles
O
O
X
X
O
Rest of PG
X_1
X_p
⋮
X


𝒪's Pasch tiles
O O X O
X O O O
O X O O
O O O X
Rest of PG
X_1
X_p
⋮
X
Untouchable by *anyone*

Summary
Anti-tic-tac-toe on Steiner triple systems

- SET = Affine Geometry: $\mathcal{X}$ wins! (Using mitres)
- Projective Geometry: $\mathcal{O}$ wins! (using Pasch configurations)
- What else is there to do?

Questions?